LINGUISTIC AND PLURILINGUAL & INTERCULTURAL COMPETENCE IN MATHEMATICS TEACHING AND LEARNING[1]

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Abstract
This article argues that mathematical competence in the sense of "mathematical literacy" (OECD 2003) includes linguistic and communicative (partial) competences. It is therefore necessary when developing competence expectations of pupils in mathematics to take into consideration previous knowledge and development in this area also. It is possible to develop checklists of cognitive-linguistic capacities on the basis of an analysis of concrete descriptions of mathematical competence underpinning mathematical educational standards (example HarmoS project in Switzerland). At the same time it also becomes clear that these capacities at least to some extent are not simply preconditions but also integral components of mathematical competence. The ability to understand and articulate in language various modes of mathematical thinking is the aim of mathematical teaching and at the same time a possible contribution of the subject to the development of language awareness and plurilingual competence.

Keywords: cognitive linguistic activities, language and communication in mathematics, language in other subjects, linguistic and mathematical competence, mathematical literacy, mathematics education and language.

[1] *I would like to thank Michael Byram for his translation into English.*

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1. INTRODUCTION

At first sight language related competences in general, and linguistic and plurilingual & intercultural competence in particular, appear to have little to do with the development of mathematical competence. If however one looks more carefully at the conceptualisation of “mathematical literacy” in the OECD/PISA studies (OECD, 2003, p. 24ff.), then it becomes clear that this concept includes, besides mathematical skills in the narrow sense, linguistic and communicative competences, necessary to understand the role which mathematics plays in the world, to make judgements based on mathematics and to take decisions; the mere mastery of basic operations and processes is not sufficient.

In this contribution we will consider the close relationship between mathematics and language related competences as this is presented in educational standards and underpinning models of competence, and as seen from the perspective of general mathematical education aims. We shall show first how and why, from this perspective, some mathematical competences should be understood in such a way that they contain language and communication related competences as integral components, and second that the promotion of such competences -- not only but also -- in mathematics teaching presents an option which arises as a consequence of the use of authentic tasks in problem resolution, and that mathematics teaching has a potential to contribute to the promotion of plurilingualism and interculturality.

In order to do this we shall first investigate the general definition of aims and the conceptualisation of mathematical literacy from the PISA studies of 2003, both of which are formulated at a high level of abstraction, with respect to presupposed language related competences, and secondly we shall reconstruct the rationale in which they are found.

In the second section descriptions of competence for mathematics will be analysed which are formulated at a middle range of abstraction, as proposed by experts for the establishment of national educational standards (Klieme, 2004) and presented in a list of cognitive linguistic activities. The educational standards for mathematics in Switzerland and the competence model from the HarmoS mathematics project which underpins them are particularly well suited for this because they contain explicit can-do formulations from which cognitive linguistic activities can be easily devised. Since this model, despite structural differences, reveals many commonalities with respect to content with other competence models (OECD, 2003; NCTM, 2000; KMK, 2004), the results will be easily transferable to these other models.

PISA test items mostly refer to more or less authentic situations from reality in which the issue is to resolve a problem with the help of mathematics. But whereas in reality such situations on the whole are real social situations, where the problem can be clarified in a group, approaches to solutions can be jointly discussed and proposals evaluated, the situations described in a paper and pencil test serve only as background scenarios; the problems are resolved by test-takers individually This means that communicative competences are not addressed. In mathematics teaching on the other hand it is possible to approach the scenario taken from reality as originally intended: problems can be analysed in class, approaches to solutions can be discussed in small groups and, either together or in individual work, tested and further developed. In the third section we shall describe two examples of a possible integration of such problem tasks in mathematics teaching where the communicative aspect is taken into proper consideration.

The ability to apply mathematical knowledge and skills not only in a specific narrow context but also in various situations and contexts in a flexible way is a central aspect of mathematical competence as conceived by PISA. For this reason, in the development of mathematical competence, what is important, in addition to the correct results and reflection on one's own process in approaching a solution, is an understanding of alternative approaches and ways of thinking of other people. The fact that behind solutions which differ only slightly in cognitive terms, there are very different perspectives, modes of thinking and strategies, only becomes evident in many cases when thinking processes are articulated in language and a sensitisation towards the finer differences between similar sounding formulations or similarly structured formula has taken place. One example of this is shown by the short video clip "Matchstick squares" discussed below. This sensitisation, thinking oneself into the solutions of other people, switching between various forms of linguistic articulation, and seeking for common linguistic means to find discursive ways to a solution, all reveal parallels to plurilingual and intercultural competence, and this will be addressed in the fourth and last part of this article.

1.1 Language and communication related aspects of 'mathematical literacy'

The general definition of aims and the conceptualisation of mathematical literacy in PISA studies have had and still have a major influence on the development of educational standards and models of competence in mathematics as a subject. For this reason it is useful to investigate these in more detail and to pay particular attention to what relationship of cognitive linguistic and social communicative competences is posited at this high level of abstraction to mathematical competences in the narrower sense, and to what structure of rationale or legitimation for this conceptualisation can be discerned. Under "cognitive linguistic activities" we shall in the following understand cognitive activities which are either inseparably linked to linguistic activities -- for example "to argue", "to name", "to judge" -- or which can be accompanied by language activities in the form of a "think aloud" process. Under "cognitive linguistic competences" we shall refer to those competences which are related to these activities. The underlying concept of competence is the one used in Klieme (2004) which in turn goes back to Weinert (2001), and which is distinctive in that it includes not only cognitive but also motivational, volitional and social components:

> As Weinert (2001, p. 27f.) puts it, competencies are "cognitive abilities and skills possessed by or able to be learned by individuals that enable them to solve particular problems, as well as the motivational, volitional and social readiness and capacity to utilise the solutions successfully and responsibly in variable situations." (Klieme, 2004, p. 17).

Based on this definition, someone is competent only when he or she, in addition to knowledge and skills, is also motivated and has the will and is ready and able, alone and in the company of others, to find appropriate problem solutions in different situations.

Under "social communicative competences" we shall understand those which are directed to discursive activities, to the negotiation of meanings and the exchange of opinions etc. and which therefore provide the basis for engagement with the values, modes and strategies of thinking of others.

The clarification of this rationale is important because it provides an indirect rationale for or against the idea of preferring mathematical literacy to other conceptualisations of mathematical competence.

With this purpose, let us look first at the definition of aims of the PISA studies in mathematics as they are formulated in the assessment framework for the 2003 PISA study, the last survey in which mathematics had a central place:

> The aim of the OECD/PISA study is to develop indicators that show how effectively countries have prepared their 15-years-olds to become active, reflective and intelligent citizens from the perspective of their uses of mathematics. To achieve this, OECD/PISA has developed assessments that focus on determining the extent to which students can use what they have learned. (OECD, 2003, p. 55)

This means that with the PISA tests two things shall be established: the effectiveness of the education system of the participating countries on the one hand and the mathematical achievements of young people on the other hand. The assessments, which were developed, were to demonstrate to what degree young people are in fact able to apply what they have learnt in mathematics lessons. The indicators founded inter alia on these assessments shall show how successful the participating countries have been in relation to mathematics in preparing their young people for their role in the world of tomorrow. The normative basis of this double definition of aims is easy to see: a right (but also a duty) of the individual to emancipation, self development and social participation, a duty (but also a right) of society to support the individual in the acquisition of the necessary competences for this. The principal idea of this normative basis is the ideal, which derives from the ideas of the Enlightenment, of a mature citizen ("active, reflective and intelligent citizens") which, provided one defines it broadly enough, includes rights of the individual to emancipation, self development and participation. Simultaneously however there arises from this the duty to form one's life oneself, to take part in societal life and to take responsibility for the development of society. These rights and duties of the individual correspond to duties and rights of society, including the duty, but also the right, to prepare young people for their role in the world of tomorrow. The education of young people as "active, reflective and intelligent citizens" is considered to be the central educational task of the school and an overarching aim of education and upbringing. Mathematics teaching and mathematical education derive their legitimacy from this aim: mathematical education is according to this conceptualisation important because and to the degree which it is an important precondition for the emancipation, self-development and societal participation of the individual as active, reflective and intelligent citizen. In content terms this is formulated at a very abstract and general level in the concept of "mathematical literacy", as follows:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2003, p. 24)

This definition of mathematical literacy is further developed in the assessment framework (p. 24ff.) and on this basis, we can say that mathematical literacy includes inter alia the following competences:

1) to put mathematical knowledge to "functional use in a multitude of different situations in varied, reflective and insight-based ways" (p.25)
2) to identify and understand the role that mathematics plays in the "natural, social and cultural setting in which the individual lives" (p. 25)
3) to make "well-founded judgements" by using mathematics (p. 24)
4) to use mathematics in ways that meet the needs of the individual's "private life, occupational life, and social life with peers and relatives, as well as life as citizen of a community". (p.25)
5) to engage with mathematics through "communicating, relating to, assessing and even appreciating and enjoying mathematics. " (p. 25)

It is clear that both cognitive linguistic and social communicative components belong to the competences, which are here formulated at a high level of abstraction. An understanding of the role which mathematics plays in the world depends both on language and on a discursive interaction with the worldview of others. In this respect what is to be understood under "world" is very broadly defined; it includes not only the world of natural sciences and technology but also the natural, social and cultural environment in which the individual lives. In addition to the epistemically framed identification and understanding of mathematical structures in the world there shall also be the action orientated capacity to make use of mathematics in various spheres of life (private, occupational, social, political). These latter spheres -- which could be complemented if necessary by others such as educational, scientific, cultural etc. -- remind us of the separation into domains in the *Common European Framework of Reference* (CEFR) (CoE, 2001, p. 10) and in particular that these spheres are each characterised by specific forms of language and communication.

Expressions such as "identify and understand" and "well founded judgements" refer to activities which have both cognitive and linguistic dimensions. One can only identify, understand and judge insofar as one relates thought and language to each other. "Knowledge" is related to activities which may well be non-verbal but which as a rule can be articulated by "thinking aloud".

To summarise: the duty of society to prepare its young people for life in the world of tomorrow and their role as "active, reflective and intelligent citizens" is assumed to be in the PISA studies a consensus of the countries participating in the investigation. If one shares the view -- irrespective of one's attitude in detail to the PISA tests, the psychometric methods, the interpretations and the presentations of results -- that this duty exists and is to be seen as the basis of the legitimation of obligatory schooling for all, then one cannot avoid recognising the social, linguistic and communicative components which are contained in the ideal of emancipation, self-development and participation. In the context of mathematics teaching this means to take into consideration in the planning of curricula and in the preparation of mathematics lessons in which real life situations or contexts mathematical competences, capacities, skills and attitudes will be used and how these must be linked to social, linguistic and communicative components. This is relevant of course not only for the subject of mathematics but for all school subjects (MSW NRW, 1999), and there have already appeared under the label "language(s) in other subjects" several studies on the platform of the Council of Europe[2] for a number of subjects [Beacco (2010) for history; Vollmer (2010) for natural sciences; Pieper (2011) for literature; Linneweber-Lammerskitten (2012) for mathematics; (Beacco, Coste, van de Ven & Vollmer, 2010b) gives a general introduction].

2. LANGUAGE AND COMMUNICATION RELATED ASPECTS OF THE HARMOS EDUCATIONAL STANDARDS FOR MATHEMATICS

The relationship between general educational aims and educational standards is not one of logical deduction, but a relationship of legitimation: educational standards cannot be derived from general educational aims but can be legitimated by them (Klieme, 2004, pp. 5, 50ff.). They make concrete the general educational aims in the form of competence expectations: “They stipulate the competencies students must possess if key learning objectives are to be considered achieved. These requirements are organised into competency models, which describe aspects, levels and the developmental trajectories of competencies.“ (Klieme, 2004, p. 16). Educational standards and the models of competence linked to them can to that extent be seen as justified insofar as they can be understood as socially acceptable concretisations of general aims of education (content legitimation) and are legally operationalised by the institutions which are responsible for them (formal legitimation). In contrast to the literacy concept of PISA, in which competences are de-

[2] *http://www.coe.int/t/dg4/linguistic/langeduc/BoxD2-OtherSub_en.asp*

scribed in a very abstract way and the ideal of the active, reflective and intelligent citizen remains legally not obligatory, national educational standards and their competence models are legally binding and a legitimation in content terms as the concretisation of the general educational aims.

In contrast to the so called “norm standards” in Germany (cf. KMK, 2004) i.e. standards that should be achieved on average the educational standards in Switzerland are conceived as "basic competences" i.e. they establish in the sense of minimal standards (Klieme, 2004, p. 22) what competences *(almost) all* pupils should master by the end of obligatory schooling. They are linked, more clearly than norm standards, normatively in two ways to rights and duties. On the one hand they formulate expectations and demands of pupils and thus make concrete the content side of obligatory schooling as an ethical-legal demand by society of the individual. On the other hand they formulate, and indeed primarily, the demand and expectation of society that it implements sufficient basic mathematical competences *for all* and thus makes concrete the ethical-legal demands of the individual for (mathematical) education.

The HarmoS competence model for mathematics (Linneweber-Lammerskitten & Wälti, 2008) was developed as a reference model for the establishment of mathematical standards of education in Switzerland by an expert consortium from the three main language and cultural regions of Switzerland. This is a multi-dimensional model in which various aspects and factors important in the description of mathematical competences are put into a systematic order. What is important for the following is above all the content dimension ("spheres of competence") and the process/action dimension ("aspects of competence"); other dimensions deal with competence level, competence development, and motivational and social elements of mathematical competence. Tables 1 and 2 show the official German, French and Italian designations and an English translation (EDK, 2011; CDIP, 2011; CDPE, 2011).

Table 1. HarmoS Mathematics Competence Model content dimension: “Competence spheres”

	Zahl und Variable	*Nombres, operations et algebra*	*Numeri e calcolo*	*Number & Variable*
1	Form und Raum	Espace	Geometria	Shape & Space
2	Funktionale Zusammenhänge	Grandeurs et measures	Grandezze e misure	Functions & Relations
3	Grössen und Masse	Fonctions	Funzioni	Size & Measurement
4	Daten und Zufall.	Analyse de données et probabilités	Dati e probabilità	Data Analysis & Probability

Table 2. HarmoS Mathematics Competence Model process and action dimension: "Competence aspects"

1	Wissen, Erkennen und Beschreiben	Savoir, reconnaître et décrire	Sapere, riconoscere e descrivere	Knowing, Recognising & Describing
2	Operieren und Berechnen	Appliquer des procédures et des techniques	Eseguire e applicare	Operating & Calculating
3	Instrumente und Werkzeuge verwenden	Utiliser des instruments et des outils	Utilizzare strumenti	Using Instruments & Tools
4	Darstellen und Kommunizieren	Présenter et communiquer	Presentare e comunicare	Presenting & communicating
5	Mathematisieren und Modellieren	Mathématiser et modéliser	Matematizzare e modellizzare	Mathematising & Modelling
6	Argumentieren und Begründen	Argumenter et justifier	Argomentare e giustificare	Arguing & Justifying
7	Interpretieren und Reflektieren der Resultate	Interpréter et analyser des résultats	Interpretare e riflettere sui risultati	Interpreting & Reflecting on Results
8	Erforschen und Explorieren	Explorer et essayer	Esplorare e tentare	Experimenting & Exploring

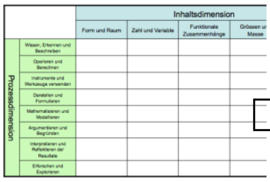
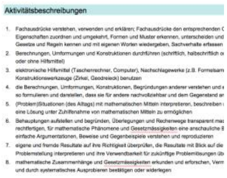
In the choice of categories, existing curricula were considered above all however already developed models of competence (KMK, 2004; NCTM, 2000; PISA 2003). The fact that designations of categories cannot always be literally translated is already an indication that the categories as such are vague[3]. For this reason there was little sense in attempting to develop cognitive linguistic activities directly from these characteristics. At the next level of concretisation the differences of meaning become notably fewer. Following a shared conviction that action and content must be considered together for the description of a mathematical competence, the categories of the action dimension were related to the content dimension and in this way a grid of 5 x 8 = 40 fields was created, initially empty, which were each used as guides (a "heuristic matrix") for descriptions of competence of a particular year group at a particular level. The fields were filled with competence descriptions in a long process of negotiation. Since these describe from a mathematical perspective[4] in complete "cando" statement forms what (all) pupils should be able to do at the

[3] *The vagueness at this high level of abstraction is not a disadvantage in the case of negotiation process. It would be necessary anyway to change to a lower level of abstraction in order to remove the vagueness. Tolerance of vagueness prevents one particular language/cultural group from dominating another or no agreement between groups being made. It is sufficient to establish clarity of concepts at the level of extraction where they are needed).*

[4] *i.e. "constructor-oriented" in the terminology of the CEFR (COE 2001, p. 39).*

end of each grade, it is relatively simple to identify the intended cognitive linguistic activities in them. Table 3 shows the development from vague categories to the summarising action descriptions.

Table 3. Development from vague categories to the summarising action descriptions

Heuristic matrix		*Competence matrix*		*Description of activities*
 Deliberately vague categories	Concretisation	 Cando-descriptions	Abstraction	 Verb forms

The following list which is based upon grade 11 (15-year-olds at the end of obligatory schooling) is abstracted from the content components and summarises similar activity descriptions:

1) understand, use and explain specialist expressions; link specialist expressions to the corresponding objects and properties and vice versa, recognise, distinguish and describe forms and patterns, know rules and laws and be able to reformulate them in one's own words, comprehend and describe substantial contents
2) carry out calculations transformations and constructions (in written form, half written or oral, with or without aids)
3) use electronic aids (pocket calculator, computer), reference works (e.g. collections of formulae), drawing tools (compass, setsquare)
4) understand the calculations, transformations, constructions and reasonings of others and formulate and present these from one's own reflection in such a way that they are understandable for others and suitable for the purpose
5) interpret, describe and model (problem) situations (of everyday life) with mathematical tools in order to make possible a solution with the aid of mathematical tools
6) make and give reasons for assertions, make reflections and calculations transparent and justify them, give a comprehensible rationale for mathematical phenomena and laws, understand and reproduce arguments proofs and counter examples

7) check the correctness of one's own and others' results, interpret the results in the light of the original problem and reflect upon their use for future problem solutions
8) investigate mathematical relationships and laws, establish assumptions and confirm or reject them through systematic testing.

Some of the infinitive constructions used are explicitly based on language/ communication competences. In particular in (1) the central issue is the relationships between the three levels of the semantic triangle: mathematical objects, properties, facts, statements, rules, operations, etc. should be grasped conceptually, but also labelled with terminology; second, mathematical concepts shall be illustrated with examples and described (in one's own words); third the meaning of specialist terminology shall be explained and made accessible with examples. In (4) it is the understanding of others and the comprehensible formulation and representation which are in focus.

There are other infinitive constructions in which competences are implicitly present: to understand, to assert, to interpret, to describe, to justify, to argue, to test, to reflect, to make assumptions, can all be done in simple cases with rudimentary language means, but on the whole more demanding language competences are needed which go far beyond the mastery of single specialist terms.

In a third group of infinitive constructions, it is indeed possible to think of acquisition by pure learning by imitation, e.g. to calculate, to transform, to construct, to use manual aids etc. However here too, from the mathematics teaching point of view, it is important to pursue meaningful learning, so that learning accompanied by language precedes mechanisation and the use of what is learnt ought to be susceptible of being complemented by "thinking aloud".

It is to be noted here that a number of expressions which refer to cognitive linguistic activities are also used outside mathematics teaching and learning, but in a mathematical context they have a special meaning. To "justify a geometric assertion" does not mean simply applying to a geometric content the activity of justification which pupils know for example from teaching in other subjects or from German language lessons. The reference to the mathematical context changes the meaning of the word "justify". The language means too which are required for such justification cannot always simply be transferred, and the same applies to strategic reflection or to the combination of normal and formal language and drawings and diagrams. This is one of the reasons why the development of language necessary for mathematics lessons cannot simply be transferred to other subjects.

3. AUTHENTIC PROBLEM TASKS IN MATHEMATICS TEACHING AND THEIR LANGUAGE AND COMMUNICATION RELATED IMPLICATIONS

Typical PISA items describe, in some cases supported by pictorial material, sketches graphs and tables, a situation in which it is necessary to resolve a problem with mathematical means or to contribute to finding a solution with the help of mathematics. The situations or contexts are varied[5]. They can come from the immediate sphere of experience of young people and be more familiar to them, or from more distant spheres and be rather unfamiliar to them. The capacity to apply mathematical knowledge and skill not only in a specific narrow context but in various situations and contexts in a flexible way is one of the central aspects of mathematical competence in the sense of the PISA definition of "mathematical literacy". Four types of situation can be identified according to their proximity or distance from the sphere of experience of learners: a personal sphere ("personal"), school, work, leisure ("educational/occupational"), neighbourhood, community, society ("public") and science and technology ("scientific") (OECD, 2003, pp. 32-34). Since the various situations are also characterised by various linguistic components (specialist terms, expressions, text types etc.) the mathematics items used in the PISA tests usually make greater demands of language and communication competences than traditional mathematical text problems. Thus it is possible to find tasks with a sales context for all four types of situation: sale of a bicycle ("personal"), daily life of a sales person in a car salesroom ("occupational"), sale of community property ("public") or a small problem of economics ("scientific"). Each of the four domains is characterised by its own linguistic and communicative demands and therefore each of the four situations makes different demands on the linguistic-communicative components of mathematical competence.

There are three further dimensions which need to be taken into consideration for the construction and categorisation of PISA tasks:

- the content dimension of the four "overarching ideas": quantity, space and shape, change and relationships, and uncertainty (ibid. 34 -- 37).
- the process dimension of the eight "characteristic mathematical competencies": thinking and reasoning, argumentation, communication, modelling, problem posing and solving, representation, using symbolic, formal and technical language and operations, and use of aids and tools (ibid. 40 -- 41).
- the dimension of the three "cognitive activities": reproduction, connections and reflection (ibid. 41ff.).

Irrespective of the kind of situation, or context, of the overarching ideas in question, of other characteristic competences and of the cognitive activities, an ideal typical mathematical problem solution in the PISA mode follows the schema of "mathematisation" (ibid. 38) presented in figure 1. This is a simple basic model, and in German-language mathematics teaching a number of more differentiated models are used alongside it. The model includes two spheres, which are metaphorically designated as different "worlds": the real world on the left and the mathematical world on the right. A problem which appears in the real world shall in the final analysis be linked to a solution in the real world. However it is often necessary to make a diversion via the mathematical world, and it is possible that the circle of

[5] *The following section is an abbreviated presentation from two essays on "task cultures" (Linneweber-Lammerskitten 2012a and 2012b).*

"mathematisation" will be travelled more than once[6]. In steps 1-3 the focus is to transform the real world problem into a mathematical problem by emphasising those characteristics which are important for a mathematical solution and leaving aside irrelevant details. In the fourth step the transformed problem is solved with mathematical means. In the fifth step the mathematical solution is transformed to the real world and tested to see if the solution in fact deals with and solves the real problem.

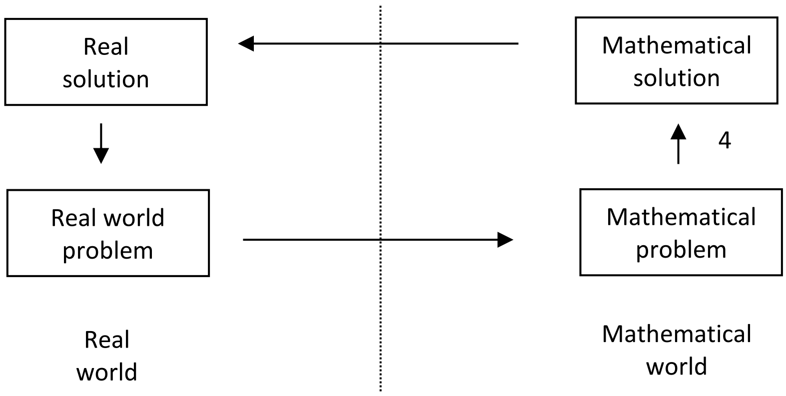


Figure 1. Ideal typical mathematical problem-solving ("mathematisation") from PISA. (Adapted from OECD, 2003, p. 38).

The model clarifies the fact that the conceptualisation of "mathematical literacy" -- like other modern conceptualisations of mathematical competence -- is based upon a broadly understood idea of mathematical knowledge and skill which notices and acknowledges the links from the real to the mathematical world and vice versa as a component of action- and problem-solving orientated mathematical activity, and is not limited to the fourth step.

If we consider as a typical example of this the PISA task "The Pizza" which with respect to the dimensions mentioned above can be linked with the situation type "personal", the overarching idea "change and relationships" (in which there are also elements of "space and shape" and "quantity"), and the cognitive activity "connection", and addresses the majority of the "characteristics of mathematical competences":

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[6] *The use of words is not uniform: one often finds alongside "mathematisation" (OECD, 2003, p. 38) "modelling", or also "(mathematical) problem-solving" where sometimes only parts of the circle are meant or specially emphasised. In the following the integration into a situation is what is decisive and corresponds to what in the KMK education standards is competence C3 "mathematical modelling" (Leiss & Blum, 2006, p. 40f.)*

> The Pizza. A pizzeria serves two round pizzas of the same thickness in different sizes. The smaller one has a diameter of 30 cm and costs 30 zeds. The larger one has a diameter of 40 cm and costs 40 zeds. Which pizza is better value for money? Show your reasoning. (OECD, 2003, p. 45)

In the first step it is necessary to develop an idea of the real situation (pizzeria, offer of two different sizes and prices) and to understand the task definition: to find out in which of the two offers one gets a better value for one's money, and to document one's thought processes on this. This means the answer should include the thought process, the calculation and the decision which one of the two pizzas is "better value for money".

In the second step it is necessary to identify the relevant data for the answer: that there are circular pizzas, that thickness plays no role, how big the two pizzas are, and what they cost. "Better value for money" should indicate that it is not a question of which pizza is cheaper but which offers "more pizza for your money", i.e. in which case the relationship of "size" to "price" is "better". In order to arrive at a correct solution it is necessary to recognise that it is not a question of the relationship of diameter to price but of surface area to price. If it were a question of the former, then the correct answer would be that both have the same "value for money". If this is recognised then it is necessary to identify, as further information required, the formula for the calculation of the area of a circle for a given diameter. A further hurdle is presented by the relation that both special offers are to be placed in: "more value for money" can be operationalised as "more pizza for the same money" or as "less money for the same amount of pizza". Both approaches can lead to the correct result but in the first case there is a comparison of two quotients of the form (area/price), and the question which of these is bigger; in the second case there is a comparison of two quotients of the form (price/area) and the question which of these is smaller.

In the third step it is necessary to bring the two parts together for a formulation of the problem in the mathematical world: the problem is now only a question of determining and comparing two quotients.

In order to reach a solution in the mathematical world, in the fourth step, calculations and comparisons have to be carried out.

In the fifth step it is necessary to test the solution found in the mathematical world, to transform it to the real world, and to reflect on its relevance.

Finally the thought process and the calculation must be presented in such a way that they are comprehensible to others.

How can we devise from this test item a learning task for the development of linguistic-communicative aspects of mathematical competence? The situation in which this task is embedded, reminds us of typical situations of talk in foreign language teaching which is also the case with other PISA test items in mathematics. This is not entirely a matter of chance, since the presentation of the embeddedness of PISA tasks in situations is strongly influenced by the *Common European Framework of Reference*. In order to complete this mathematisation circle a number of cognitive activities are necessary (OECD, 2003, p. 39) which are of cognitive-linguistic character. This becomes clear if one verbalises these activities with the help of language patterns:

- "What is mathematically relevant in this problem is not .... but..., perhaps it is also important, that...."
- "perhaps we will advance further if we formulate the problem thus…. We can understand the issue in this way ..., i.e. we could also try...."
- "we can express something more formally thus..., or represent with a formula thus..."
- "the problem has a similar structure to the xy-problem which we have already solved".

Instead of formulating the problem of "better value for money" at a rather impersonal level as in the test task, one could in the classroom set the exercise of acting in a role-play corresponding to a conversational situation in a pizzeria, and make use of these language patterns. What would a corresponding situation for an "active, reflective and intelligent citizen" look like in reality? It would be:

- action orientated instead of epistemologically orientated:

  "Which pizza shall I order?"

- discursive -- embedded in a communicative situation:

  "Which are you going to choose?"

- with justification at various levels:

  "I will take the large one because it is cheaper" OR "I'll take the smaller one because I am less hungry" OR "I'll take the smaller one because I think a pizza for 40zeds is too expensive".

Utterances of this kind -- if necessary stimulated by the teacher -- can provoke reflection on the two relations of comparison "more pizza for the same money" and "less money for the same amount of pizza". For, in fact, one can order neither the one nor the other, but has a choice between a large pizza which costs more than the small one and the small one which costs less than the big one. Embedded in the discourse with other pupils it becomes clear that the question of "better value for money" is, in a society operating according to laws of the market, an important but not the only criterion for rational decision-making.

The "literacy" conception has not only influenced the development of educational standards and models of competence in German-speaking countries but has also led to an increased construction, analysis and empirical research on corresponding "authentic" learning tasks. The following now well-known task from Leiß can serve as an example. It was in fact conceived for mathematics teaching but could be used, with some different emphases, as a learning task in natural sciences teaching or in language teaching:

> Mr Stein lives in Trier, 20 km from the Luxembourg border. He drives to Luxembourg to fill up his VW Golf and there is a petrol station immediately after the frontier. There a litre of petrol costs only €1.05, as opposed to €1. 30 in Trier. Is the journey worthwhile for Mr Stein? Given reasons for your answer. (Leiß & Blum, 2006, p. 42)

The context of the problem is authentic because in fact residents of villages near the frontier make use of the possibility of buying less expensively in the neighbouring country and of filling up their cars more cheaply. For a citizen who is interested and critically aware there are also other questions, for example: "Is it reasonable? Is it worthwhile? Are there similar possibilities for me? How would I decide?" and yet more. This task differs from traditional mathematical tasks in several ways:

- there are data missing but necessary for the calculation of costs (for example the capacity of the tank, petrol consumption, position of petrol stations in Luxembourg and in Trier)
- it is not a matter of calculating profit or loss, but of comparing and evaluating two alternatives for action
- the criterion for the evaluation of the chosen alternative ("Is it worthwhile for...") requires interpretation
- the necessary rationale does not (only) consist of a justification of a calculation but also of an explanation of the assumptions on which the decision rests and in what constraining conditions they exist
- the task brings up a number of further questions and problems
- the task shows (perhaps with further reflection) the possibilities but also the limits of mathematical problem-solving.

These points are precisely more or less typical for problem-solving situations in reality and must be taken into consideration in the construction of a competence which will correspond to the overarching educational games mentioned above:

- when a problem arises in the real world not all information is "delivered" which is necessary or helpful for the solution, but must be sought out, or perhaps assumptions must be made and simplifications undertaken
- normally in problem-solving situations decisions between action alternatives are required and for this it is important to know the pros and cons of the alternatives
- the criteria on the basis of which the decisions must be made are themselves often in need of clarification and interpretation
- decisions must to a certain extent be comprehensible for others or at least for oneself
- part of a strategy of problem-solving in the real world is reflection about problems already solved -- on the one hand retrospectively and on the other hand prospectively -- to see if the solution which has been found can be improved, whether it can be generalised or transferred to other domains, or whether the assumptions which have been made are appropriate etc
- problems in the real world are often multilayered, and must be observed from various sites and from various perspectives. The responsibility for decisions for

action cannot be handed over to a scientific discipline, but remains with the actor.

In learning tasks of this kind, which are characterised by proximity to reality, openness to questioning, demands for decision-making and justification, missing information on the one hand and irrelevant information on the other hand, there is a major potential relationship to overarching educational aims. However, these are learning tasks which make high demands of learners and of teachers.

Let us now think about the problem "filling up the tank" from another perspective. Because of its proximity to reality, the task not only creates interest in a solution and in the course of the attempt to find a solution stimulates a readiness for reflection, but it can also contribute to a positive image of mathematics and thereby to motivation which may also be activated outside school. What is however decisive here is that the use of the task in the classroom is successful and does not lead to demands which are too high for weak learners or to frustration among strong ones. Let us therefore consider what steps are important in understanding and resolving the task.

First it is necessary to develop from the descriptive information given an appropriate idea of the real situation as Mr Stein encounters it, and causes him to travel more than 40 km to fill up his car. Secondly the task must be understood in the way it relates to the real situation: one must decide whether Mr Stein's journey is worthwhile and the answer must be justified. For this it is necessary to have already an idea of the kind of answer and of the form in which the answer is expected: "According to the assumptions that I have made the journey for Mr Stein is/is not worthwhile because ....".

Already in this first step learners might fail because of the language difficulties or because of a lack of geographical knowledge. In order to counteract this it would be possible to provide the learners with a roadmap, further pictorial and text material, or to have them work on the scenario in pairs or groups and thus to ensure they understand the situation and the task. It remains open however what exactly "It is worthwhile" should mean -- the first understanding could be that "it is cheaper to fill up the car in Luxembourg".

In the second phase details have to be identified which could be mathematically relevant (the two prices for petrol, the distance to the border). Missing data must also be identified, or information acquired or assumptions made. The irrelevant data or data which are no longer needed for the process of resolving the task can be put aside, for example what kind of car it is, because this is not important in itself but only with respect to petrol consumption or the capacity of the tank etc. Here learners could profitably work together on an approach to the solution but may be reliant upon stimulation and structured help from the teacher concerning the missing data. They ought however to arrive independently at insights such as "It depends on how many litres the tank contains" etc., and establish this information themselves or make reasonable assumptions.

In the third step the real world problem has to be transformed into a mathematical problem in which in the final analysis only two alternatives for costing appear, in the form of comparisons. In this transformation that which is mathematically irrelevant is abstracted and also the problem horizon is limited: Time spent and other additional cost factors, but also ecological, national economic and ethical aspects remain unconsidered. They have to be considered in the transformation back from the mathematical solution into a real-world solution. Perhaps some pupils will at this point realise that these and other points of view play a part but have to be put aside initially for the calculation to be done. The creation of two comparisons is a further stage which has to be ensured before learners can begin to carry out the calculations.

The calculations to be done are simple, in a simplified model, since in the comparisons there are only three basic types of calculation used. They lead to the result that approximately €8 can be saved in Luxembourg. It is interesting but not without risk for the process of teaching to ask whether the journey is worthwhile for Mr Stein. For here the saving has to be put in relation to the time spent. Furthermore, the costs which are left out of the modelling (wear and tear on tyres, consumption of oil, reduction of the value of the car etc) need to be thought about. Perhaps a more refined model has to be created. Environmentally conscious pupils will have doubts concerning the effect on the environment, consumption of scarce resources, the increase in traffic, the nuisance to residents etc. On the one hand these thoughts are particularly important with respect to the overarching learning aims, but on the other hand, learners should not go home with the impression that they have carried out calculations but that that "was not the correct answer after all". It would also be fatal to have a split between a solution for the mathematics classroom and another for the real world. It would be important that learners realised, over a longer process, that a problem in the real world usually has to be considered from various perspectives. Cost saving is one aspect which has to be established and perhaps can be made more precise by refining the model. The question about the relationship of time and money can perhaps be investigated with "what if"-questions: “What if Mr Stein lived 40 miles away and the saving was one euro?"

It is clear that the success of such a lesson depends not only on the potential of the learning task as such but also on its didactic and methodological embedding in the teaching.

It is also interesting to think about variations of such a task, in particular with respect to its subject specialism background and its possible ideological significance. The following variation was used with the help of a roadmap and was to be solved by learners working individually.

> Mr Stein lives in Trier not far from the Luxembourg border. He drives his VW golf to Luxembourg to fill up the tank and there is a little station just beyond the frontier. There a litre of petrol costs only €0.85 because of the lower mineral oil taxes. It is always good to economise, because one never knows what one might need the money for later. But does Mr Stein really save money and if he does how much does he save? (Sinusmonatsaufgabe February 2006)

The intention is on the one hand to create a stronger demand for independence of learners: in comparison to the original task the explicit information about distance and about the cost of petrol in Trier is left out, and these need to be found from the appropriate information sources or complemented by estimates. On the other hand the problem is reduced, by the narrower questioning and stimulus "Does Mr Stein save money?" and "Economising is always good", to a pure exercise in calculation, which was no doubt the intention here. What is worse is that with a sentence like “It is always good to economise, because one never knows what one might need the money for later.“ an ideological value attitude is transposed without discussion which is not in accord with the overarching educational aims promoting maturity and societal participation. A more open variation of the task and a discursive processing of the problem with phases of group work is surely to be preferred.

4. THE POTENTIAL OF MATHEMATICS TEACHING IN THE DEVELOPMENT OF PLURILINGUAL AND INTERCULTURAL COMPETENCES

The concepts "plurilingual" and "intercultural" are currently used in various shades of meaning and are not always distinguished clearly from related words such as "multilingual" and "pluricultural" (Dervin, 2010). In the following section I will rely on the definitions in Beacco and Byram (2007, p. 114f.):

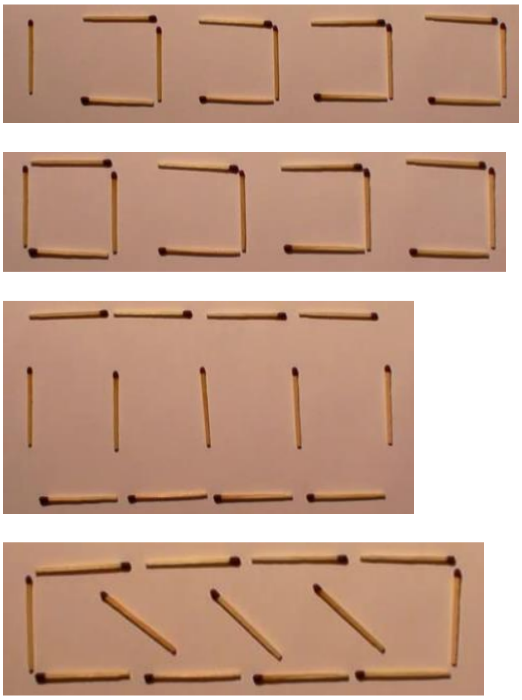
> Plurilingual (competence): capacity to successively acquire and use different competences in different languages, at different levels of proficiency and for different functions. The central purpose of plurilingual education is to develop this competence.
>
> Intercultural competence: combination of knowledge, skills, attitudes and behaviours which allow a speaker, to varying degrees, to recognise, understand, interpret and accept other ways of living and thinking beyond his or her home culture. This competence is the basis of understanding among people, and is not limited to language ability.

Starting from general educational aims and ideals such as those of the mature citizen, of emancipation, of self-development and participation, it is possible to justify a right to plurilingual and intercultural education (Coste, Cavalli, Crișan, & van de Ven, 2009) analogous to the right to mathematics education, and to legitimise projects to implement plurilingualism and intercultural education (Cavalli, Coste, Crișan, & van de Ven, 2009). The competences which are created in this way are doubtless not integral components of mathematical competence but they do contain a kernel – understanding and valuing otherness, alternative points of view, ways of thinking and other strategies, the change of perspective switching between language systems etc -- which is also important for successful engagement with mathematics in and outside the school. I would like in the following to pursue this and show that mathematics teaching possesses a strong potential to contribute something through the development of mathematical literacy to the construction of plurilingual and intercultural competence. It can be noted in passing that mathematics teaching can in turn profit from the implementation of plurilingualism and intercultural education.

The ability to use mathematical knowledge and skills not only in a specific narrow context but also in various situations and contexts in a flexible way, is as already noted one of the central aspects of mathematical competence according to the PISA definition of "mathematical literacy". Also following Weinert's definition competence involves, in addition to "cognitive abilities and skills", the "motivational, volitional and social readiness and capacity to utilise the solutions successfully and responsibly in variable situations" (in: Klieme, 2004, p. 70). In order to acquire this flexibility it is important that pupils do not only learn standard solutions in pure and applied mathematics, but are also confronted with problems from various domains of application).

Furthermore, in addition to variation of contexts, situations and domains in prescribed tasks we see increasing importance attached in contemporary mathematics teaching to the variation of problems by learners themselves (cf. Linneweber-Lammerskitten, 2012b and literature cited there). This variation can be made by changes in the formulation of the problem, the path to the solution and/or the results. Thus for example by departing from an already resolved problem a new task could be to change the context, the situation, the domain, the numerical values or the linguistic formulation of the problem, whereas the solution process and the result *mutatis mutandis* remain the same. Another task could be to start with results which are numerically different and/or transposed to other contexts, situations or domains, and then find appropriate problems und approaches leading to these solutions. Finally the approaches can be varied and the task can consist of (i) seeking alternatives to approaches which have already been found or (ii) finding as many different approaches to solving the problem or (iii) finding pathways to solve the problem more easily or (iv) finding approaches which open new points of view on the problem. The video clip "matchstick squares" on YouTube[7] makes clear how one can see the same starting position in different ways and from there arrive at different (even though algebraically equivalent) solutions by different paths. The point in this clip is to deconstruct a figure made up of four squares from 13 matches in such a way that it becomes clear what the principle of construction is and then find a formula with which one can calculate the number of matches necessary for any figure of n squares.

[7] *http://www.youtube.com/watch?v=aG0m5FX0TmY&feature=plcp*



1 + 4x3
Tn = 3n + 1

4 + (4-1)x3
Tn = 3(n-1) + 4

4 + 4 + (4+1)
Tn = 2n + (n+1)

(4 + 4 + 2) + (4-1)
Tn = (2n + 2) + (n-1)

Figure 2. Matchstick squares.

The initial figure is perceived differently each time a different principle of construction is noted and – perhaps after several intermediate steps – expressed with another formula which still reveals the traces of the principle of construction or the "way of seeing" in the final figure. In the first case one initial match is separated and four figures follow from three matches. In the second case one square of four matches is the starting figure followed by (4-1) figures of three matches. In the third case four upper and four lower horizontal matches are noted which join (4+1) perpendicular matches. In the fourth case the figure is understood as a rectangle which is surrounded by 2x4+2 matches and is subdivided in the interior by (n -1) matches. Further variations are conceivable -- what point of view could for example lie behind the formula Tn=4n-(n-1)x1?

The video clip makes clear from one example that from the same initial situation there can be various perspectives, which may lead to different approaches and results which are different in form but all have their justification. If one speaks with learners about this then it is usually evident that the points of view of learners do indeed differ from each other. Perhaps some self-confident pupils think initially that their own way of thinking is the better because it is their own, and other (failure orientated) pupils think their own is worse, precisely because it is theirs. A discussion of similar problems in group work will however lead to the insight that taking into consideration various perspectives, contributing one's own suggestions and taking into consideration other ways of seeing and thinking increase the chances of success. Whereas intercultural competence is directed to cultural otherness, the focus here is on individual otherness in ways of seeing and thinking, which should be "recognised", "understood", "interpreted" and "accepted". In both cases competence includes a combination of knowledge, skills, attitudes and modes of behaviour which allow pupils to engage with otherness in an appropriate way.

It is not surprising to find that most of the basic rules which have been formulated for intercultural education (Byram, Gribkova, & Starkey, 2002, p. 25) are relevant here too, i.e. linked to individual otherness:

- Participants are expected to listen to each other and take turns.
- Where a discussion is chaired, the authority of the chair is respected.
- Even heated debates must be conducted in polite language.
- Discriminatory remarks, particularly racist, sexist and homophobic discourse and expressions, are totally unacceptable at any time.
- Participants show respect when commenting on and describing people portrayed in visuals or texts.
- All involved have the responsibility to challenge stereotypes.
- A respectful tone is required at all times.

The understanding and the ability and readiness to become involved with the otherness of modes of seeing and thinking includes the languages used for this. In this way what has been said above about interculturality can be extended or transposed to the language domain. Furthermore mathematics itself can be understood as a multilingual system which presupposes, for a genuine understanding, the capacity for flexible change between algebraic formula language, mathematical specialist language, language of teaching and one's own modes of speaking and thinking, and thus to a certain extent presupposes plurilingualism competence and language awareness. Thus although plurilingual and intercultural competence are not integral components of mathematical competence, there are nonetheless some parallels between them so that one can hope that the development of the one is simultaneously the development of the other.

5. CONCLUSION

Mathematical competence in the sense of "mathematical literacy" includes linguistic and communicative (partial) competences. Competence expectations based on this which are made of pupils must therefore take into account prior knowledge and development in this domain. It is possible to derive checklists of cognitive-linguistic activities from the analysis of concrete descriptions of mathematical competence underpinning educational standards in mathematics (example of the HarmoS project). It is also clear from this that these capacities are not only preconditions but also integral components of mathematical competence. Authentic problem-solving tasks reveal their potential in mathematics teaching only when cognitive-linguistic and social-communicative aspects are taken into consideration. To be able to understand different routes of mathematical thinking and to articulate them is the aim of mathematics teaching and at the same time the possible contribution of the subject to the development of plurilingual and intercultural competence.

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